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APPROXIMATE METHOD FOR ABSORPTION AT HIGH TEMPERATURES

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It was shown in a previous study that at elevated temperatures the absorption of a radiating gas is significantly affected by overlapping "hot bands" [1]. The final theoretical relations are, however, complex which complicates the calculations. Furthermore, the extension of the results to other problems and considerations; e.g. non-isothermal transport, is also made correspondingly difficult. In the present note a simplified model is proposed and tested.

The complexity of the basic relations is a direct consequence of the inclusion of the hot bands into the analysis. To avoid this difficulty Edwards [2] considered a single equivalent band and used an average or equivalent value for the line width, b_A, or the broadening parameter, $\beta_A = 2\pi b_A/d$, where d is the line spacing. The relation for β_A was determined from the nonoverlapped strong line condition which corresponds to the maximum influence of the line broadening parameter. This procedure may also be used in conjunction with our previous studies.

From equation (A.5) of [1] we have that the total band absorptance, corresponding to the above condition, is given by

$$\frac{A}{F_P + F_R} = (8\beta/\pi)^{\frac{1}{2}} \sum_v (u_v)^{\frac{1}{2}} = (8\beta X/d\pi)^{\frac{1}{2}} \sum_v (S_{0,v})^{\frac{1}{2}} \qquad (1)$$

where S_0 is a characteristic line intensity, X is the pressure pathlength, F is a characteristic band width, the subscripts P and R refer to the P and R branches, and the optical depth $u_v = S_{0,v}X/d$. In terms of an equivalent band we have (cf. equation (B.4) of [3])

$$\frac{A}{F_P + F_R} = (8\beta_A u_v/\pi)^{\frac{1}{2}} = (8\beta_A X/d\pi)^{\frac{1}{2}} (\sum_v S_{0,v})^{\frac{1}{2}}. \qquad (2)$$

Equating equations (1) and (2) yields

$$(\beta_A/\beta)^{\frac{1}{2}} = \sum_v (S_{0,v})^{\frac{1}{2}}/(\sum_v S_{0,v})^{\frac{1}{2}} \qquad (3)$$

which agrees with the result obtained by Edwards [2]. Now, from [3] we have that

$$S_{0,v} = D_1 \alpha_v^{v+1} (hcB_e/kT)^{\frac{1}{2}} \qquad (4)$$

and from [2] or [4]

$$a_v^{v+1} = \frac{8\pi^3 N_T \nu B^2}{3hcQ_{\text{vib}}} (1 - e^{-hc\nu/kT}) e^{-hc\nu(v+\frac{1}{2})/kT} \qquad (5)$$

where B^2 is the square of the electric dipole matrix element, Q_{vib} is the vibrational partition function and N_T is the total number density. Combining the above relations and using the result that B^2 varies like $(v + 1)$ [2, 4] yields

$$\left(\frac{\beta_A}{\beta}\right)^{\frac{1}{2}} = \frac{\sum_{v=0}^{\infty} [(v+1) e^{-vhc\nu/kT}]^{\frac{1}{2}}}{[\sum_{v=0}^{\infty} (v+1) e^{-vhc\nu/kT}]^{\frac{1}{2}}}. \qquad (6)$$

Now, using the Euler–Maclaurin summation formula and the identities [5]

$$Y = \sum_{v=0}^{\infty} e^{-vhc\nu/kT} = [1 - e^{-hc\nu/kT}]^{-1}$$

$$Y^{n+1} = \sum_{v=0}^{\infty} \frac{(v+n)!}{n!\,v!} e^{-vhc\nu/kT}$$

we obtain

$$\beta_A(T)/\beta(T) = [1 + (hc\nu/kT)^{-\frac{3}{2}} e^{hc\nu/2kT} \Gamma(\tfrac{3}{2}, hc\nu/2kT)]^2 (1 - e^{-hc\nu/kT}) \qquad (7)$$

where $\Gamma(a, b)$ is the incomplete Gamma function [6].

The total band absorptance may now be directly calculated, using, for example, the expressions given in Appendix B of [3] with β_A determined from equation (7). A comparison of the present results with the experimental data of Tien and Giedt [7] and Abu-Romia and Tien [8] for carbon monoxide is shown in Table 1 along with the correlation results [2, 8–10]. The agreement is seen to be very good. For completeness we emphasize that the value of $\beta_A(T)$ was obtained *a priori* as given by equation (7). Another value of $\beta_A(T)$ may be obtained empirically from the experimental data. For carbon monoxide the two values for $\beta_A(T)$ differ by less than 2 per cent over the temperature range from 300°K to 1800°K.

Table 1. Comparison of theoretical and experimental results for the total band absorptance for the 4·7μ *band of carbon monoxide*

Temperature (°K)	Effective pressure P_e (atm)	Pressure pathlength PX (atm-cm)	Experimental data (Tien and Giedt [7], Abu-Romia and Tien [8]) (cm^{-1})	Present results (cm^{-1})	Two band results [1] (cm^{-1})	Correlation result [2, 8–10] (cm^{-1})
900	1·02	1·0	27·7	35·21	30	36·9
	2·04	2·0	57·6	71·12	60·3	66·5
	3·06	3·0	90·5	103·19	102·4	89·5
	0·51	2·5	38·0	40·44	39·7	41·1
	1·02	5·0	82·0	80·18	82·3	86·2
	2·04	10	134·3	141·54	124·5	149·4
	3·06	15·0	191·0	188·25	168	196·0
	0·255	2·5	26·3	26·7	29·4	25·0
	0·51	5·0	55·0	56·47	69·7	59·5
	1·02	10·0	106·2	111·55	114	118·5
	2·04	20·0	176·2	178·49	160·5	191·3
	3·06	30·0	234	230·08	210·2	240·0
	0·255	5·0	37·8	40·44	39·7	37·5
	0·511	10·0	66·0	78·78	67·5	90·0
	1·02	20·0	134·0	142·23	124·5	153·0
	2·04	40·0	226·0	223·11	202·3	227·0
	3·06	60·0	273·0	278·88	258·0	285·5
1800	1·02	10·0	129	117·33	97·7	107
	1·53	15·0	172	168·6	145·5	160
	2·04	20·0	195	208·05	192·3	208
	2·55	25·0	247	242·56	238·5	246
	3·06	30·0	278	270·16	278·0	277
	1·02	20·0	159	157·76	139·8	154
	1·53	30·0	221	216·92	202	224
	2·04	40·0	268	262·28	264	278
	2·55	50·0	317	299·74	316	311
	3·06	60·0	347	335·24	368·5	340

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SHAPE OF POROUS COOLED REGION FOR SURFACE HEAT FLUX AND TEMPERATURE BOTH SPECIFIED

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NOMENCLATURE*

C_p,	specific heat at constant pressure;
G,	mass flow rate of coolant per unit length normal to x, y plane;
k_m,	effective thermal conductivity of porous region;
l,	position along boundary S;
M	for incompressible flow $M = \frac{1}{2}\frac{\mu}{\mu_\infty}\frac{t_\infty}{t}$; for compressible flow $M = \mu/\mu_\infty$;
P,	for incompressible flow $P = p/p_\infty$; for compressible flow $P = (p/p_\infty)^2$;
p,	pressure;
Q_{total},	heat conducted into surface S per unit length normal to x, y plane;
T,	temperature ratio t/t_∞;
t,	absolute temperature;
$\mathbf{u}$,	velocity vector;
X, Y,	dimensionless coordinates $x/h_r, y/h_r$, where h_r is defined in equation (8);
α,	surface absorptivity for incident radiation;
κ,	permeability of porous material;
λ,	parameter $\rho_\infty C_p \kappa p_\infty / 2\mu_\infty k_m$;
μ,	fluid viscosity;
$\tilde{\nabla}$,	dimensionless gradient in X, Y plane, $\hat{\imath}(\partial/\partial X) + \hat{\jmath}(\partial/\partial Y)$.

Subscripts

i,	insulated and impervious surface;
r,	reference condition;
s,	on surface where coolant exits from porous medium;
0,	on surface where coolant enters porous medium.

INTRODUCTION

AN EFFECTIVE cooling technique can be obtained by utilizing porous materials so that coolant can be forced out through the surface that is subjected to a heat load. For a given pumping pressure the amount of coolant passing through a location on the cooled surface will depend on the flow resistance along the path through the porous region to that location. The shape of the porous region will regulate the flow resistance and hence the local cooling capability along the surface. If a heat flux distribution is imposed along the surface and it is also desired to maintain the surface at a uniform design temperature, it is required to know what shape the porous region should have to meet these conditions.

In [1] a heat transfer analysis was devised for a three-dimensional porous cooled region of specified shape. The analysis obtained the region temperature distribution in terms of a potential function found by solving Laplace's equation in the geometry of the porous region subject to simple boundary conditions. Since steady-state heat

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* Other symbols are defined in text.